National Semiconductor

AF150
Universal Wideband Active Filter

General Description

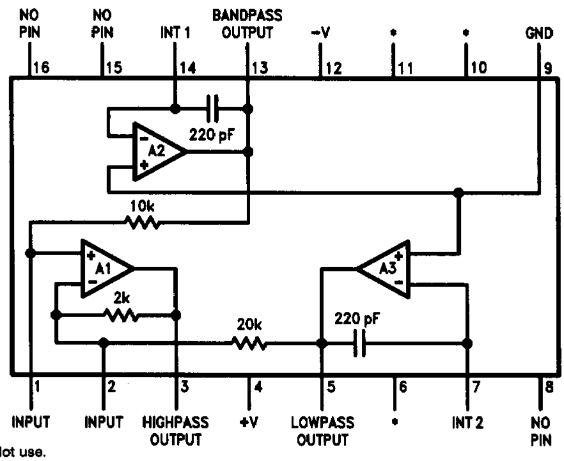
The AF150 wideband active filter is a general second order lumped RC network. Only four external resistors are required to program the AF150 for specific second order functions. Low pass, high pass and band pass functions are available simultaneously at separate outputs. Notch and all pass functions can be formed by summing the outputs with an external amplifier. Higher order filters are realized by cascading AF150 active filters with appropriate programming resistors.

Any of the classical filter configurations, such as Butterworth, Bessel, Cauer and Chebyshev can be formed.

Features

- Independent Q, frequency, gain adjustments
- Low sensitivity to external component variation
- Separate low pass, high pass, band pass outputs
- Inputs may be differential, inverting or non-inverting
- All pass and notch outputs may be formed
- Operates to 100 kHz
- Q range to 500
- Power supply range ±5V to ±18V
- High accuracy ±1% unadjusted
- Q frequency product 2×10^5

Connection Diagram



***Note:** Internally connected. Do Not use.

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Top View

Ceramic Dual-In-Line Package
Order Number AF150-1CJ or AF150-2CJ
See NS Package Number HY13A

Absolute Maximum Ratings

If Military/Aerospace specified devices are required, please contact the National Semiconductor Sales Office/Distributors for availability and specifications.

Supply Voltage	±18V
Power Dissipation (Note 1)	900 mW/Package (500 mW/Amp)
Differential Input Voltage	±36V
Output Short-Circuit Duration (Note 1)	Infinite
Operating Temperature	−25°C to +85°C
Storage Temperature	−25°C to +100°C
Lead Temperature (Soldering, 10 Seconds)	300°C

Electrical Characteristics

Specifications apply for $V_S = \pm15V$, over −25°C to +85°C unless otherwise specified.

Symbol	Parameter	Conditions	Min	Typ	Max	Units
	Frequency Range	$f_c \times Q \leq 2 \times 10^5$			100k	Hz
	Q Range				500	Hz/Hz
	f_o Accuracy AF150-1J AF150-2J	$f_c \times Q \leq 5 \times 10^4$, $T_A = 25°C$			 ±2.5 ±1.0	%
$\Delta f_o/\Delta T$	f_o Temperature Coefficient			±50	±150	ppm/°C
	Q Accuracy	$f_c \times Q \leq 5 \times 10^4$, $T_A = 25°C$			±7.5	%
$\Delta Q/\Delta T$	Q Temperature Coefficient			±300	±750	ppm/°C
PSRR	Power Supply Rejection Ratio		80	100		dB
CMRR	Common Mode Rejection		80	100		dB
I_{OS}	Input Offset Current	$T_j = 25°C$		3	50	pA
I_B	Input Bias Current	$T_j = 25°C$		30	200	pA
V_{CM}	Input Common-Mode Voltage Range	$V_S = \pm15V$	±11	±12		V
I_S	Power Supply Current	$V_S = \pm15V$, $T_A = 25°C$		15	30	mA

Note 1: Any of the amplifier's outputs can be shorted to ground indefinitely; however, more than one should not be simultaneously shorted as the maximum package power dissipation will be exceeded.

Applications Information

CIRCUIT DESCRIPTION AND OPERATION

A schematic of the AF150 is shown in *Figure 1*. Amplifier A1 is a summing amplifier with inputs from integrator A2 to the non-inverting input and integrator A3 to the inverting input.

By adding external resistors the circuit can be used to generate the second order transfer function:

$$T(s) = \frac{a_3 s^2 + a_2 s + a_1}{s^2 + b_2 s + b_1}$$

The denominator coefficients determine the complex pole pair location and the quality of the poles where

$$\omega_o = \sqrt{b_1} = \text{the radian center frequency}$$

$$Q = \frac{\omega_o}{b_2} = \text{the quality of the complex pole pair}$$

If the output is taken from the output of A1, numerator coefficients a_1 and a_2 equal zero, and the transfer function becomes:

$$T(s) = \frac{a_3 s^2}{s^2 + \frac{\omega_o}{Q}s + \omega_o^2} \quad \text{(high pass)}$$

If the output is taken from the output of A2, numerator coefficients a_1 and a_3 equal zero and the transfer function becomes:

$$T(s) = \frac{a_2 s}{s^2 + \frac{\omega_o}{Q}s + \omega_o^2} \quad \text{(band pass)}$$

If the output is taken from the output of A3, numerator coefficients a_3 and a_2 equal zero and the transfer function becomes:

$$T(s) = \frac{a_1}{s^2 + \frac{\omega_o}{Q}s + \omega_o^2} \quad \text{(low pass)}$$

Using an external op amp and the proper input and output connections, the circuit can also be used to generate the transfer functions for a notch and all pass filter.

In the transfer function for a notch function a_2 becomes zero, a_1 equals ω_z^2 and a_3 equals 1. The transfer function becomes:

$$T(s) = \frac{s^2 + \omega_z^2}{s^2 + \frac{\omega_o}{Q}s + \omega_o^2} \quad \text{(notch)}$$

Applications Information (Continued)

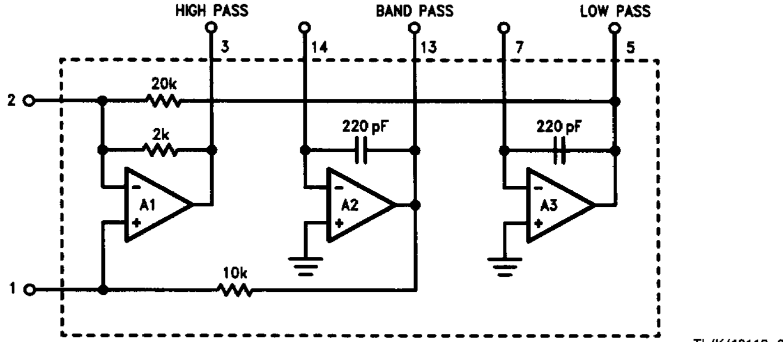


FIGURE 1. AF150 Schematic

In the all pass transfer function $a_3 = 1$, $a_2 = -\omega_o/Q$ and $a_1 = \omega_o^2$. The transfer function becomes:

$$T(s) = \frac{s^2 - \frac{\omega_o}{Q}s + \omega_o^2}{s^2 + \frac{\omega_o}{Q}s + \omega_o^2} \quad \text{(all pass)}$$

The relationships between the generalized coefficients and the external resistors will be found in the appendix. It is not, however, necessary to use these theoretical, if not "messy", equations to solve for the proper external resistor values. In general, it is assumed that the user has knowledge of the frequency and Q of the specific filter he is designing. For higher order filters of various types, the reader is directed to any of the available texts on filters (see bibliography) for information and tables concerning the location of the poles and zeros. Once the specifics of the filter are found from the tables, it is simply a matter of cascading the sections with proper attention to some general guidelines which are included later in the application section.

The following discussion gives a step-by-step procedure for designing filters with several examples given for clarity.

FREQUENCY TUNING

Two equal value frequency setting resistors are required for frequencies above 1 kHz. For lower frequencies, T tuning or the addition of external capacitors is required. Using external capacitors allows the user to go as low in frequency as he desires. T tuning and external capacitors can be used together.

Two resistor tuning for 1 kHz to 100 kHz:

$$R_f = \frac{228.8 \times 10^6}{f_o}\,\Omega \quad (1)$$

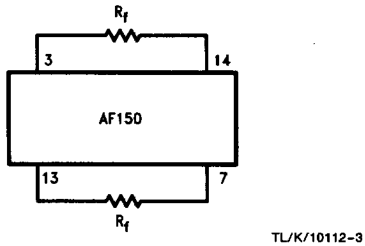
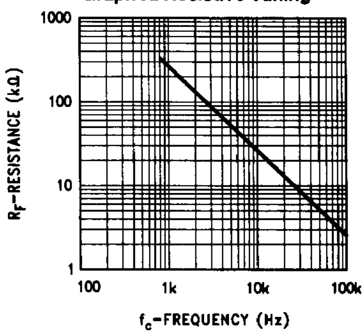


FIGURE 2. Resistive Tuning

Graph A. Resistive Tuning



T resistive tuning for $f_o < 1$ kHz:

$$R_S = \frac{{R_T}^2}{R_f - 2\,R_T} \quad (2)$$

R_f from equation 1.

Applications Information (Continued)

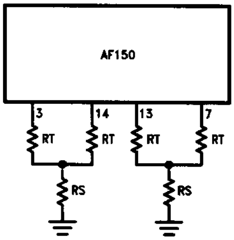


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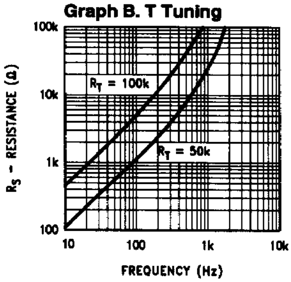
FIGURE 3. T Tuning

Graph B. T Tuning



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If external capacitors are used for $f_o < 1$ kHz, then equation 3 should be used.

$$R_f = \frac{0.05033}{f_o\,(C + 220 \times 10^{-12})}\,\Omega \qquad (3)$$

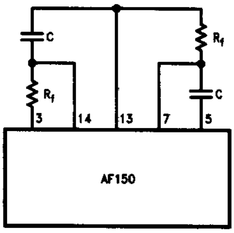

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FIGURE 4. Low Frequency RC Tuning

Q DETERMINATION

Setting the Q requires one resistor from either pin 1 or pin 2 to ground. The value of the Q setting resistor depends on the input connection and input resistance as well as the value of the Q. The Q will be inversely proportional to the resistance from pin 1 to ground and directly proportional to resistance from pin 2 to ground.

NON-INVERTING CONNECTION*

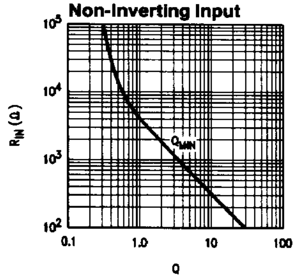
To determine the Q resistor, choose a value of input resistor, R_{IN} (*Figures 5* and *6*) and calculate Q_{MIN} (Graph C).

$$Q_{MIN} = \frac{1 + \frac{10^4}{R_{IN}}}{3.48}$$

***Note:** The discussion of "non-inverting" and "inverting" has to do with the phase relationship between the input port and the *low pass* output port. Refer to *Figure 1* for other output port phase relationships.

If the Q required in the circuit is greater than Q_{MIN}, use the circuit configuration shown in *Figure 5* and equation 4 to calculate R_Q, the Q resistor. If the Q of the circuit is less than Q_{MIN}, use the circuit configuration shown in *Figure 6* and equation 5.

Graph C. Q_{MIN} Non-Inverting Input



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For Q > Q_{MIN} in non-inverting mode:

$$R_Q = \frac{10^4}{3.48Q - 1 - \frac{10^4}{R_{IN}}}\,\Omega \qquad (4)$$

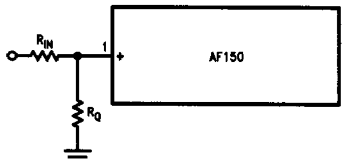

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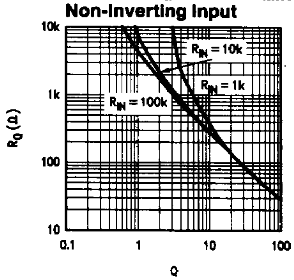
FIGURE 5. Q Tuning for Q > Q_{MIN}, Non-Inverting Input

Graph D. R_Q for Q > Q_{MIN}, Non-Inverting Input



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For Q < Q_{MIN} in non-inverting mode:

$$R_Q = \frac{2 \times 10^3}{0.3162\,\frac{\left(1 + \frac{10^4}{R_{IN}}\right)}{Q} - 1.1}\,\Omega \qquad (5)$$

Applications Information (Continued)

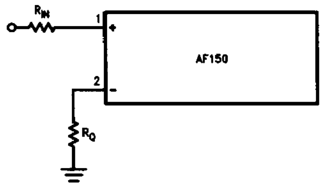

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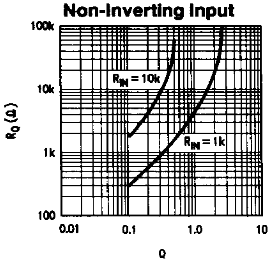
FIGURE 6. Q Tuning for $Q < Q_{MIN}$, Non-Inverting Input

Graph E. R_Q for $Q < Q_{MIN}$, Non-Inverting Input



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INVERTING CONNECTION*

For any Q in inverting mode:

$$R_Q = \frac{10^4}{3.16Q\left(1.1 + \frac{2 \times 10^3}{R_{IN}}\right) - 1}\,\Omega \qquad (6)$$

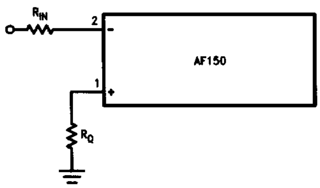

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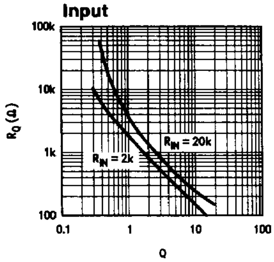
FIGURE 7. Q Tuning, Inverting Input

Graph F. Q Tuning Inverting Input



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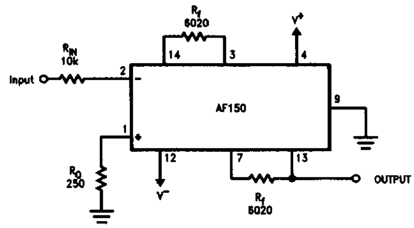
***Note:** The discussion of "non-inverting" and "inverting" has to do with the phase relationship between the input port and the *low pass* output port. Refer to *Figure 1* for other output port phase relationships.

DESIGN EXAMPLE

Non-Inverting Band Pass Filter

Center frequency 38 kHz = f_O, 10 Hz/Hz = Q, 10k = R_{IN}.



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Using Equation 1

$$R_f = \frac{228.8 \times 10^6}{f_O}\,\Omega$$

$$R_f = \frac{228.8 \times 10^6}{38 \times 10^3} = 6020\Omega$$

Using Equation 6

$$R_Q = \frac{10^4}{3.16Q\left(1.1 + \frac{2 \times 10^3}{R_{IN}}\right) - 1}\,\Omega$$

$$R_Q = 250\Omega$$

From equation 33, the center frequency gain is found to be 6.3 V/V (16 dB). If the center frequency gain is to be adjusted, equation 33 can be solved for R_Q in terms of R_{IN} and this substituted into equation 6 to find the required R_{IN} and R_Q.

NOTCH FILTERS

Notches can be generated by two simple methods: using RC input *(Figure 8)* or low pass/high pass summing *(Figure 9)*. The RC input method requires adding a capacitor to pin 14 and a resistor connects to pin 7. The summing method requires two resistors connected to the low pass and high pass output.

The difference between the two possible methods of generating a notch is that the capacitor connection requires a high quality precision capacitor and the gain of the circuit is difficult to adjust because the gain and zero location are both dependent on C_Z and R_Z. The amplifier summing method requires 3 precision resistors and an external operational amplifier. However, the gain can be adjusted independent of the notch frequency.

For input RC notch tuning:

$$R_Z = \frac{C_Z R_f \times 10^{12}}{220}\left(\frac{f_O}{f_Z}\right)^2\Omega \qquad (7)$$

f_Z = frequency of notch (zero location)

Applications Information (Continued)

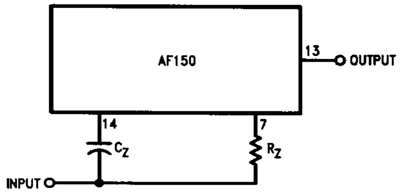

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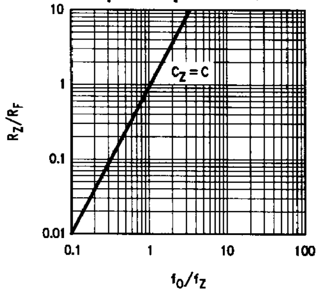
FIGURE 8. Input RC Notch

Graph G. Input RC Notch



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For the low pass/high pass summing technique,

$$R_h = \left(\frac{f_Z}{f_o}\right)^2 \frac{R_L}{10} \qquad (8)$$

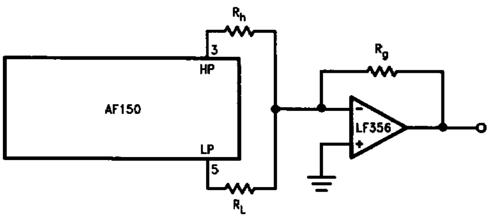

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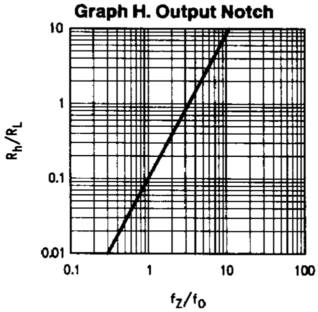
FIGURE 9. Output Notch

Graph H. Output Notch



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DESIGN EXAMPLE

19 kHz notch using RC input.

Center frequency	19 kHz	f_o
Zero frequency	19 kHz	f_Z
	20	Q

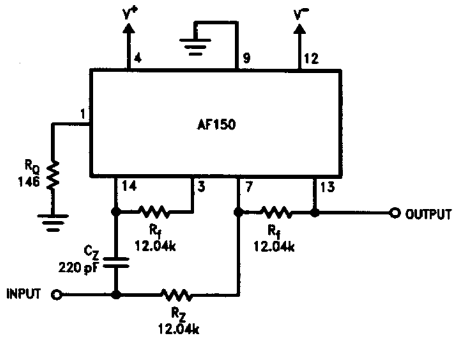

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FIGURE 10. RC Notch, 19 kHz

Using equation 1:

$$R_f = \frac{228.8 \times 10^6}{f_o}\,\Omega$$

$$R_f = 12{,}040\Omega$$

Using equation 4 with $R_{IN} = \infty$:

$$R_Q = \frac{10^4}{3.48Q - 1 - \dfrac{10^4}{R_{IN}}}\,\Omega$$

$$R_Q = 146\Omega$$

Using equation 7:

$$R_Z = \left(\frac{C_Z\,R_F \times 10^{12}}{220}\right)\left(\frac{f_o}{f_Z}\right)^2 \Omega$$

$$R_Z = 12{,}040\Omega$$

DESIGN EXAMPLE

19 kHz notch using low pass/high pass summing

Center frequency	19 kHz	f_o
Zero frequency	19 kHz	f_Z
	20	Q

Using equation 1:

$$R_f = \frac{228.8 \times 10^6}{f_o}\,\Omega$$

$$R_f = 12{,}040\Omega$$

Using equation 4, choose $R_{IN} = 10\ k\Omega$:

$$R_Q = \frac{10^4}{3.48Q - 1 - \dfrac{10^4}{R_{IN}}}\,\Omega$$

$$R_Q = 148\Omega$$

Using equation 8:

$$R_h = \left(\frac{f_Z}{f_o}\right)^2 \frac{R_L}{10}$$

Choose $R_L = 20k$, then $R_h = 2k$

Applications Information (Continued)

TRIALS, TRIBULATIONS AND TRICKS

Certainly, there is no substitute for experience when applying active filters, working with op amps or riding a bicycle. However, the following section will discuss some of the finer points in more detail, and hopefully alleviate some of the fears and problems that might be encountered.

TUNING TIPS

In applications where 2 to 3% accuracy is not sufficient to provide the required filter response, the AF150 stages can be tuned by adding trim pots or trim resistors in series or parallel with one of the frequency determining resistors and the Q determining resistor.

When tuning a filter section, no matter what output configuration is to be used in the circuit, measurements are made between the input and the band pass (pin 13) output.

Before any tuning is attempted the low pass (pin 5) output should be checked to see that the output is not clipping. At the center frequency of the section the low pass output is 10 dB higher than the band pass output and 20 dB higher than the high pass. This should be kept in mind because if clipping occurs the results obtained when tuning will be incorrect.

Frequency Tuning

By adjusting the resistance between pins 7 and 13 the center frequency of a section can be adjusted. If the input is through pin 1 the phase shift at center frequency will be 180° and if the input is through pin 2 the phase shift at center frequency will be 0°. Adjusting center frequency by phase is the most accurate but tuning for maximum gain is also correct.

Q Tuning

The Q is tuned by adjusting the resistance between pin 1 or pin 2 and ground. Low Q tuning resistors will be from pin 2 to ground ($Q < 0.6$). High Q tuning resistors will be from pin 1 to ground. To tune the Q correctly, the signal source must have an output impedance very much lower than the input resistance of the filter since the input resistance affects the Q. The input must be driven through the same resistance the circuit will see to obtain precise adjustment.

The lower 3 dB (45°) frequency, f_L, and the upper 3 dB (45°) frequency, f_H, can be calculated by the following equations:

$$f_H = \left(\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1}\right) \times (f_o)$$

$$f_L = \left(\sqrt{\left(\frac{1}{2Q}\right)^2 + 1} - \frac{1}{2Q}\right) \times (f_o)$$

where f_o = center frequency

When adjusting the Q, set the signal source to either f_H or f_L and adjust for 45° phase change or a 3 dB gain change.

Notch Tuning

If a circuit has a jw axis zero pair the notch can be tuned by adjusting the ratio of the summing resistors (low pass/high pass summing) or the input resistance (input RC).

In either case the signal is connected to the input and the proper resistor is adjusted for a null at the output.

Special Cases

When using the input RC notch the unit cannot be tuned through the normal input so an additional 100k resistor can be added at pin 1 and the unit can be tuned normally. Then the 100k input resistor should be grounded and the notch tuned through the normal RC input.

An alternative way of tuning is to tune using the Q resistor as the input. This requires the Q resistor be lifted from ground and connecting the signal source to the normally grounded end of the Q resistor. This has the problem that when the Q resistor is grounded after tuning, its value is decreased by the output impedance of the source. This technique has the advantage of not requiring an additional resistor.

TUNING PROCEDURE

Center Frequency Tuning

Set oscillator to center frequency desired for the filter section, adjust amplitude and check that clipping does not occur at the low pass output pin 5.

Adjust the resistance between pins 13 and 7 until the phase shift between input and band pass output is 180°.

Q Tuning

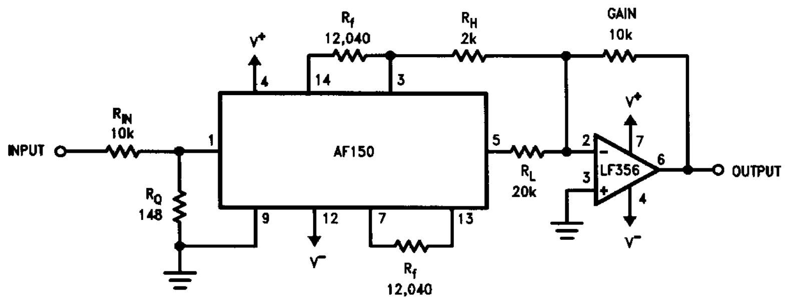
Set oscillator to upper or lower 45° frequency (see tuning tips) and tune the Q resistor until the phase shift is 135° (upper 45° frequency) or 225° (lower 45° frequency).

Zero Tuning

Set the oscillator output to the zero frequency and tune the zero resistor for a null at the output of the summing amplifier.

FILTER DESIGN

Since most filter tables are in terms of a normalized low pass prototype, the filter to be designed is usually reduced to a low pass prototype. After the low pass transfer function is found, it is transformed to obtain the transfer function for the actual filter desired. The low pass amplitude response which can be defined by four quantities, defined below:



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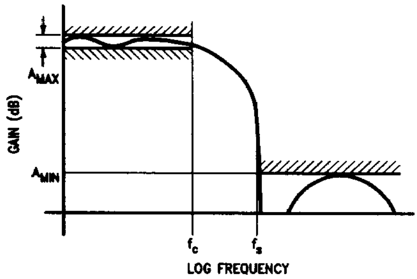
Applications Information (Continued)

Low Pass Response



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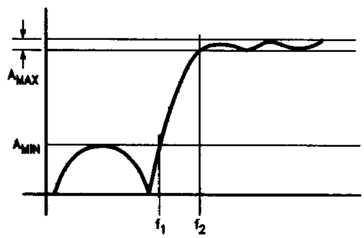
A_{MAX} = the maximum peak-to-peak ripple in the pass band
A_{MIN} = the minimum attenuation in the stop band
f_c = the pass band cutoff frequency
f_s = the stop band start frequency

By defining these four quantities for the low pass prototype the normalized pole and zero locations and the Q (quality) of the poles can be determined from tables or by computer programs.

To obtain the high pass from the low pass filter tables, A_{MAX} and A_{MIN} are the same as for the low pass case, but $f_c = 1/f_2$ and $f_s = 1/f_1$.

High Pass Response



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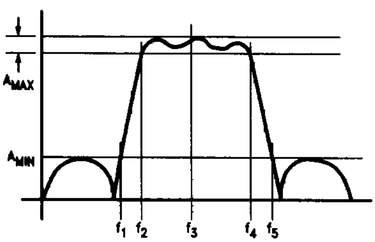
To obtain the band pass from the low pass filter tables, A_{MAX} and A_{MIN} are the same as for the low pass case, but:

$$f_c = 1 \qquad f_s = \frac{f_5 - f_1}{f_4 - f_2}$$

where $f_3 = \sqrt{f_1 \times f_5} = \sqrt{f_2 \times f_4}$ i.e., geometric symmetry

$f_5 - f_1 = A_{MIN}$ bandwidth
$f_4 - f_2$ = Ripple bandwidth

Band Pass Response



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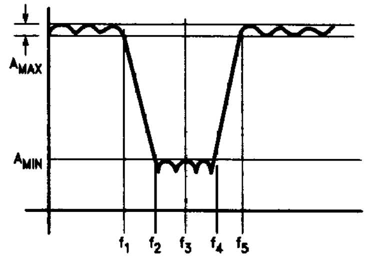
To obtain the notch from the low pass filter tables, A_{MAX} and A_{MIN} are the same as for the low pass case and

$$f_c = 1, \qquad f_s = \frac{f_5 - f_1}{f_4 - f_2}$$

where $f_3 = \sqrt{f_1 \times f_5} = \sqrt{f_2 \times f_4}$

Notch Response



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Normalized Low Pass Transformed to Un-Normalized Low Pass

The normalized low pass filter has the pass band edge normalized to unity. The un-normalized low pass filter instead has the pass band edge at f_c. The normalized and un-normalized low pass filters are related by the transformation $s = s\omega_c$. This transforms the normalized pass band edge $s = j$ to the un-normalized pass band edges $s = j\omega_c$.

Normalized Low Pass Transformed to Un-Normalized High Pass

The transformation that can be used for low pass to high pass is $S = \omega_C/s$. Since S is inversely proportional to s, the low frequency and high frequency responses are interchanged. The normalized low pass $1/(S^2 + S/Q + 1)$ transforms to the un-normalized high pass.

$$\frac{s^2}{s^2 + \frac{\omega_C}{Q} s + \omega_C^2}$$

Normalized Low Pass Transformed to Un-Normalized Band Pass

The transformation that can be used for low pass to band pass is:

$$S = \frac{s^2 + \omega_0^2}{BS \times s}$$

where ω_0^2 is the center frequency of the desired band pass filter and BW is the ripple bandwidth.

Normalized Low Pass Transformed to Un-Normalized Band Stop (Or Notch)

The bandstop filter has a reciprocal response to a band pass filter. Therefore, a bandstop filter can be obtained by first transforming the low pass prototype to a high pass and then performing the band pass transformation.

SELECTION OF TRANSFER FUNCTION

The selection of a function which approximates the shape of the response desired is a complicated process. Except in the simplest case, it requires the use of tables or computer

Applications Information (Continued)

programs. The form of the transfer function desired is in terms of the pole and zero locations. The most common approximations found in tables are Butterworth, Chebychev, Elliptic and Bessel. The decision as to which approximation to use is usually a function of the requirements and system objectives. Butterworth filters are the simplest but have the disadvantage of requiring high order transfer functions to obtain sharp roll-offs.

The Chebychev function is a min/max approximation in the pass band. This approximation has the property that it is equiripple which means that the error oscillates between maximums and minimums of equal amplitude in the pass band. The Chebychev approximation, because of its equiripple nature, has a much steeper transition region than the Butterworth approximation.

The elliptic filter, also known as Cauer or Zolotarev filters, are equiripple in the pass band and stop band and have a steeper transition region than the Butterworth or the Chebychev.

For a specific low pass filter three quantities can be used to determine the degree of the transfer function: the maximum pass band ripple, the minimum stop band attenuation, and the transition ratio ($tr = \omega_s/\omega_c$). Decreasing A_{MAX}, increasing A_{MIN}, or decreasing t_r will increase the degree of the transfer function. But for the same requirements the elliptic filter will require the lowest order transfer function. Tables and graphs are available in reference books such as "Reference Data for Radio Engineers", Howard W. Sams & Co., Inc., 5th Edition, 1970 and Erich Christian and Egon Eisenmann, "Filter Design Tables and Graphs"; John Wiley and Sons, 1966.

For specific transfer functions and their pole locations such texts as Louis Weinberg, "Network Analysis and Synthesis", McGraw Hill Book Company, 1962 and Richard W. Daniels, "Approximation Methods for Electronic Filter Design", McGraw-Hill Book Company, 1974, are available.

DESIGN OF CASCADED MULTISECTION FILTERS

The first step in designing is to define the response required and define the performance specifications:

1. Type of filter:

   Low pass, high pass, band pass, notch, all pass

2. Attenuation and frequency response
3. Performance

   Center frequency/corner frequency plus tolerance and stability

   Insertion loss/gain plus tolerance and stability

   Source impedance

   Load impedance

   Maximum output noise

   Power consumption

   Power supply voltage

   Dynamic range

   Maximum output level

The second step is to find the pole and zero location for the transfer function which meet the above requirements. This can be done by using tables and graphs or network synthesis. The form of the transfer function which is easiest to convert to a cascaded filter is a product of first and second order terms in these forms:

First Order	Second Order	
$\frac{K}{s + \omega_r}$	$\frac{K}{s^2 + \frac{\omega_o}{Q}s + \omega_o^2}$	(low pass)
$\frac{Ks}{s + \omega_r}$	$\frac{Ks^2}{s^2 + \frac{\omega_o}{Q}s + \omega_o^2}$	(high pass)
	$\frac{Ks}{s^2 + \frac{\omega_o}{Q}s + \omega_o^2}$	(band pass)
	$\frac{K(s^2 + \omega_z^2)}{s^2 + \frac{\omega_o}{Q}s + \omega_o^2}$	(notch)
	$\frac{s^2 - \frac{\omega_o}{Q}s + \omega_o^2}{s^2 + \frac{\omega_o}{Q}s + \omega_o^2}$	(all pass)

Each of the second order functions is realizable by using an AF150 stage. By cascading these stages the desired transfer function is realized.

CASCADING SECOND ORDER STAGES

The primary concern in cascading second order stages is to minimize the difference in amplitude from input to output over the frequencies of interest. A computer program is probably required in very complicated cases but some general rules that can be used that will usually give satisfactory results are:

1. The highest Q pole pair should be paired with the zero pair closest in frequency.
2. If high pass and low pass stages are cascaded, the low pass sections should be the higher frequency and high pass sections the lower frequency.
3. In cascaded filters of more than two sections, the first section should be the section with Q closest to 0.707 and then additional stages should be added in order of least difference between first stage Q and their Q.

DESIGN EXAMPLES OF CASCADE CONNECTIONS

Example 1:

Consider a 4th order Butterworth low pass filter with a 10 kHz cutoff (−3 dB) frequency and input impedance ≥30 kΩ.

From tables, the normalized filter parameters are:

F1 = 1.0 Q1 = 0.541

F2 = 1.0 Q2 = 1.306

Thus, relative to the design required

F1 = (1.0) (10 kHz) = 10 kHz

F2 = (1.0) (10 kHz) = 10 kHz

Section 1

F = 10 kHz, Q = 1.306

$$R_f = \frac{228.8 \times 10^6}{f_o}\,\Omega \qquad \text{(Using equation 1)}$$

$R_f = 22{,}880\Omega$

Applications Information (Continued)

Select input resistor 31.6 kΩ

$$Q_{MIN} = \frac{1 + \frac{10^4}{R_{IN}}}{3.48}$$

$$Q_{MIN} = 0.378$$

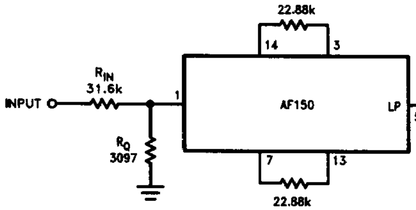
Thus, $Q > Q_{MIN}$

Therefore:

$$R_Q = \frac{10^4}{3.48Q - 1 - \frac{10^4}{R_{IN}}} \Omega \text{ (Using equation 4)}$$

$$R_Q = 3097\Omega$$

First Stage



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Section 2

$$f_o = 10k, \quad Q = 0.541$$

Since f_o is the same as for the first section:

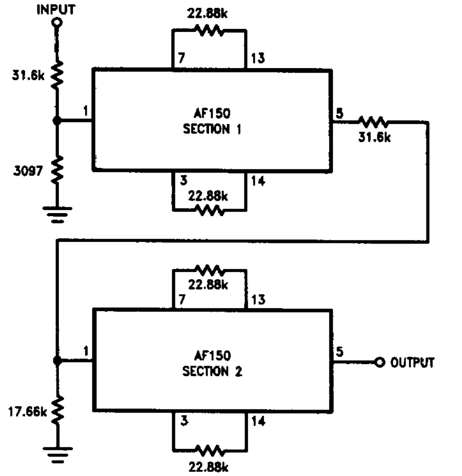
$$R_f = 22.88\ k\Omega$$

Select $R_{IN} = 31.6\ k\Omega$

$$R_Q = \frac{10^4}{3.48Q - 1 - \frac{10^4}{R_{IN}}} \Omega \text{ (Using equation 4)}$$

$$R_Q = 17{,}661\Omega$$

Complete Filter, Example 1



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Example 2.

Consider the design of a low pass filter with the following performance:

$$f_c = 10\text{ kHz}$$
$$f_s = 11\text{ kHz}$$
$$A_{MAX} = 1\text{ dB}$$
$$A_{MIN} = 40\text{ dB}$$

It is found that a 6th order elliptic filter will satisfy the above requirements. The parameters of the design are:

STAGE	f_o (kHz)	Q	f_z (kHz)
1	5.16	0.82	29.71
2	8.83	3.72	13.09
3	10.0	20.89	11.15

Applications Information (Continued)

Stage 1

a) From equation 1, R_F is found to be 44.34k

b) From equation 4, R_Q is found to be 11.72k assuming R_{IN} (arbitrary) is 10 kΩ.

To create the transmission zero f_z, at 29.71 kHz, use equation 8.

$$R_h = \left(\frac{f_z}{f_o}\right)^2 \frac{R_L}{10}, \quad \text{or } R_h = \left(\frac{29.71}{5.16}\right)^2 \frac{R_L}{10}$$

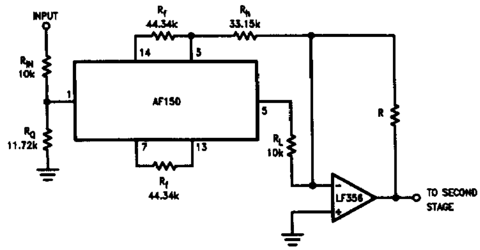
Thus,

$$R_h = 3.315\, R_L$$

If R_L is arbitrarily chosen as 10 kΩ, R_h = 33.15k.

Thus, the design of the first stage is:

First Stage



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where the feedback resistor, R, around the external op amp may be used to adjust the gain.

Stage 2

The second stage design follows exactly the same procedure as the first stage design. The results are:

a) From equation 1, R_f = 25.91k

b) From equation 4, R_Q = 913.6Ω, again assuming R_{IN} is arbitrarily 10k.

c) $R_h = \left(\frac{13.09}{8.83}\right)^2 \frac{R_L}{10}$ or $R_h = 0.22\, R_L$

Selecting R_L = 10k, then R_h = 2.2k, the second stage design is shown below.

Stage 3

The third stage design, again, is identical to the first 2 stages and the results are (for R_{IN} = 10k):

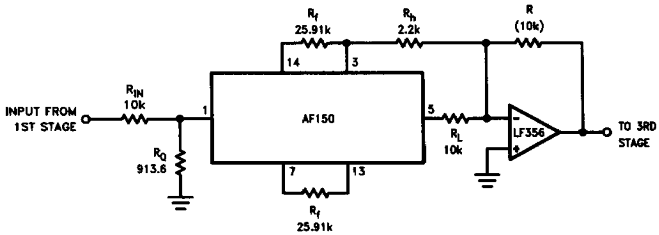
$$R_f = \frac{228.8 \times 10^6}{f_o} = 22.88\text{k}$$

$$R_Q = \frac{10^4}{3.48Q - 1 - \frac{10^4}{R_{IN}}} = 141.4\Omega$$

$$R_h = \left(\frac{f_z}{f_o}\right)^2 \frac{R_L}{10} = \left(\frac{11.5}{10}\right)^2 \frac{R_L}{10} \qquad R_h = 0.124\, R_L$$

Let R_L = 20k, R_h = 2.48k

Second Stage



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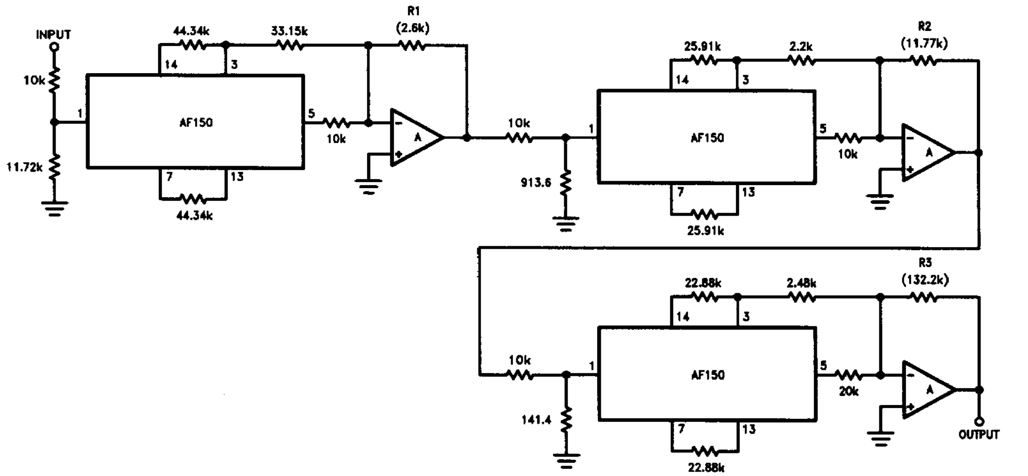
Applications Information (Continued)

Filter for Example 2



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Note 1: Select R1, R2, R3 for desired gain.

Note 2: All amplifiers LF356.

From equation 13, the DC gain of the first section is

$$A_{V1} = \frac{11}{1 + \frac{R_{IN}}{10^4} + \frac{R_{IN}}{R_Q}}$$

$$A_{V1} = \frac{11}{1 + \frac{10^4}{10^4} + \frac{10^4}{11.72 \times 10^3}} = 3.86 \text{ V/V}$$

Similarly, the DC gain of the second and third sections are:

$$A_{V2} = 0.850$$

$$A_{V3} = 0.151$$

Therefore, the overall DC gain is 0.495 and can be adjusted by selecting R1 with respect to 10k, R2 with respect to 10k or R3 with respect to 20k.

Applications Information (Continued)

For convenience, a standard resistor value table is given below.

Standard Resistance Values are obtained from the Decade Table by multiplying by multiples of 10. As an example, 1.33 can represent 1.33Ω, 133Ω, 1.33 kΩ, 13.3 kΩ, 133 kΩ, 1.33 MΩ.

Standard 5% and 2% Resistance Values

Ohms	Ohms	Ohms	Ohms	Ohms	Ohms	Ohms	Ohms	Ohms	Ohms	Ohms	Megohms	
10	27	68	180	470	1,200	3,300	8,200	22,000	56,000	150,000	0.24	0.62
11	30	75	200	510	1,300	3,600	9,100	24,000	62,000	160,000	0.27	0.68
12	33	82	220	560	1,500	3,900	10,000	27,000	68,000	180,000	0.30	0.75
13	36	91	240	620	1,600	4,300	11,000	30,000	75,000	200,000	0.33	0.82
15	39	100	270	680	1,800	4,700	12,000	33,000	82,000	220,000	0.36	0.91
16	43	110	300	750	2,000	5,100	13,000	36,000	91,000		0.39	1.0
18	47	120	330	820	2,200	5,600	15,000	39,000	100,000		0.43	1.1
20	51	130	360	910	2,400	6,200	16,000	43,000	110,000		0.47	1.2
22	56	150	390	1,000	2,700	6,800	18,000	47,000	120,000		0.51	1.3
24	62	160	430	1,100	3,000	7,500	20,000	51,000	130,000		0.56	1.5

Decade Table Determining ½% and 1% Standard Resistance Values

1.00	1.21	1.47	1.78	2.15	2.61	3.16	3.83	4.64	5.62	6.81	8.25
1.02	1.24	1.50	1.82	2.21	2.67	3.24	3.92	4.75	5.76	6.98	8.45
1.05	1.27	1.54	1.87	2.26	2.74	3.32	4.02	4.87	5.90	7.15	8.66
1.07	1.30	1.58	1.91	2.32	2.80	3.40	4.12	4.99	6.04	7.32	8.87
1.10	1.33	1.62	1.96	2.37	2.87	3.48	4.22	5.11	6.19	7.50	9.09
1.13	1.37	1.65	2.00	2.43	2.94	3.57	4.32	5.23	6.34	7.68	9.31
1.15	1.40	1.69	2.05	2.49	3.01	3.65	4.42	5.36	6.49	7.87	9.53
1.18	1.43	1.74	2.10	2.55	3.09	3.74	4.53	5.49	6.65	8.06	9.76

Appendix (See Footnote)

The specific transfer functions for some of the most useful circuit configurations using the AF150 are illustrated in *Figures 11–17*. Also included are the gain equations for each transfer function in the frequency band of interest, the Q equation, center frequency equation and the Q determining resistor equation. Q_{MIN} is a function of R_{IN} (see Graph C).

a. Non-Inverting Input *(Figure 10)*
Transfer Equations are:

$$\frac{e_h}{e_{IN}} = \frac{s^2\left[\dfrac{1.1}{1 + \dfrac{R_{IN}}{10^4} + \dfrac{R_{IN}}{R_Q}}\right]}{\Delta} \text{ (high pass)} \quad (9)$$

$$\frac{e_b}{e_{IN}} = \frac{-s\,\omega_1\left[\dfrac{1.1}{1 + \dfrac{R_{IN}}{10^4} + \dfrac{R_{IN}}{R_Q}}\right]}{\Delta} \text{ (band pass)} \quad (10)$$

$$\frac{e_\ell}{e_{IN}} = \frac{\omega_1\,\omega_2\left[\dfrac{1.1}{1 + \dfrac{R_{IN}}{10^4} + \dfrac{R_{IN}}{R_Q}}\right]}{\Delta} \text{ (low pass)} \quad (11)$$

where

$$\Delta = s^2 + s\left[\frac{1.1}{1 + \dfrac{10^4}{R_Q} + \dfrac{10^4}{R_{IN}}}\right]\omega_1 + 0.1\,\omega_1\,\omega_2 \quad (12)$$

$$\left.\frac{e_\ell}{e_{IN}}\right|_{s \to 0} = \frac{11}{\left(1 + \dfrac{R_{IN}}{10^4} + \dfrac{R_{IN}}{R_Q}\right)} \text{ (DC Gain)} \quad (13)$$

$$\left.\frac{e_h}{e_{IN}}\right|_{s \to \infty} = \frac{1.1}{\left(1 + \dfrac{R_{IN}}{10^4} + \dfrac{R_{IN}}{R_Q}\right)} \text{ (High Freq. Gain)} \quad (14)$$

$$\left.\frac{e_b}{e_{IN}}\right|_{\omega = \omega_0} = -\frac{\left(1 + \dfrac{10^4}{R_Q} + \dfrac{10^4}{R_{IN}}\right)}{\left(1 + \dfrac{R_{IN}}{10^4} + \dfrac{R_{IN}}{R_Q}\right)} \text{ (Center Freq. Gain)} \quad (15)$$

$$\omega_1 = \frac{10^{12}}{R_{f1} \times 220} \qquad \omega_2 = \frac{10^{12}}{R_{f2} \times 220}$$

where

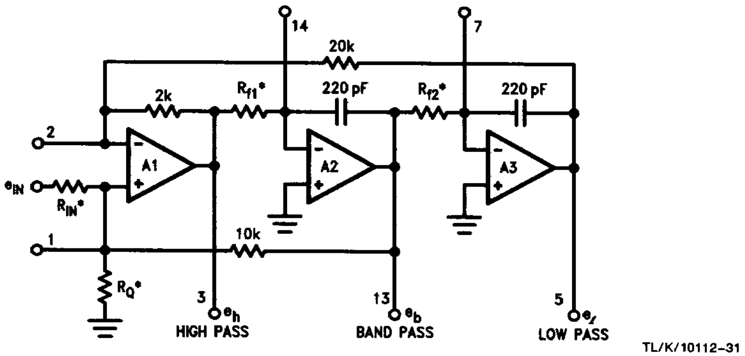
$$\omega_0 = \sqrt{0.1\,\omega_1\,\omega_2}, \text{ (see Footnote)}$$

$$Q = \left(\frac{1 + \dfrac{10^4}{R_{IN}} + \dfrac{10^4}{R_Q}}{1.1}\right)\sqrt{0.1\left(\frac{\omega_2}{\omega_1}\right)} \quad (16)$$

$$R_Q = \frac{10^4}{\left(\dfrac{1.1Q}{\sqrt{0.1\dfrac{\omega_2}{\omega_1}}}\right) - 1 - \dfrac{10^4}{R_{IN}}} \quad (17)$$

Note: It should be noted that in the text of this paper, ω_1 and ω_2 have been assumed equal, and hence $R_{f1} = R_{f2}$. No generality is lost in this assumption and it facilitates the design. However, for completeness, the equations given are exact.

Appendix (Continued)



*External Components

FIGURE 11. Non-Inverting Input ($Q > Q_{MIN}$)

b) Non-inverting input *(Figure 12)* transfer equations are:

$$\frac{e_h}{e_{IN}} = \frac{s^2\left[\dfrac{1.1 + \dfrac{2 \times 10^3}{R_Q}}{1 + \dfrac{R_{IN}}{10^4}}\right]}{\Delta} \text{ (high pass)} \qquad (18)$$

$$\frac{e_b}{e_{IN}} = \frac{-s\,\omega_1\left[\dfrac{1.1 + \dfrac{2 \times 10^3}{R_Q}}{1 + \dfrac{R_{IN}}{10^4}}\right]}{\Delta} \text{ (band pass)} \qquad (19)$$

$$\frac{e_\ell}{e_{IN}} = \frac{\omega_1\,\omega_2\left[\dfrac{1.1 + \dfrac{2 \times 10^3}{R_Q}}{1 + \dfrac{R_{IN}}{10^4}}\right]}{\Delta} \text{ (low pass)} \qquad (20)$$

where

$$\Delta = s^2 + s\,\omega_1\left[\frac{1.1 + \dfrac{2 \times 10^3}{R_Q}}{1 + \dfrac{10^4}{R_{IN}}}\right] + 0.1\,\omega_1\,\omega_2 \qquad (21)$$

$$\left.\frac{e_\ell}{e_{IN}}\right|_{s \to 0} = \frac{1.1 + \dfrac{2 \times 10^3}{R_Q}}{0.1\left(1 + \dfrac{R_{IN}}{10^4}\right)} \qquad (22)$$

$$\left.\frac{e_h}{e_{IN}}\right|_{s \to \infty} = \frac{1.1 + \dfrac{2 \times 10^3}{R_Q}}{1 + \dfrac{R_{IN}}{10^4}} \qquad (23)$$

$$\left.\frac{e_b}{e_{IN}}\right|_{\omega = \omega_0} = -\frac{1 + \dfrac{10^4}{R_{IN}}}{1 + \dfrac{R_{IN}}{10^4}} \qquad (24)$$

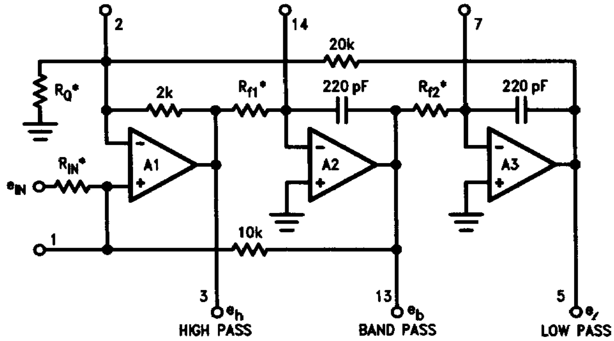
$$\omega_1 = \frac{10^{12}}{R_{f1} \bullet 220}, \quad \omega_2 = \frac{10^{12}}{R_{f2} \bullet 220}$$

$$\omega_0 = \sqrt{0.1\,\omega_1\,\omega_2}$$

$$Q = \left[\frac{1 + \dfrac{10^4}{R_{IN}}}{1.1 + \dfrac{2 \times 10^3}{R_Q}}\right]\sqrt{0.1\,\frac{\omega_2}{\omega_1}} \qquad (25)$$

$$R_Q = \frac{2 \times 10^7}{\left(1 + \dfrac{10^4}{R_{IN}}\right)\left(\dfrac{\sqrt{0.1\,\dfrac{\omega_2}{\omega_1}}}{Q}\right) - 1.1} \qquad (26)$$

Appendix (Continued)



*External Components

FIGURE 12. Non-Inverting Input ($Q < Q_{MIN}$)

c) Inverting input *(Figure 13)* transfer function equations are:

$$\frac{e_h}{e_{IN}} = \frac{-s^2\left(\frac{2 \times 10^3}{R_{IN}}\right)}{\Delta} \text{ (high pass)} \qquad (27)$$

$$\frac{e_b}{e_{IN}} = \frac{s\,\omega_1\left(\frac{2 \times 10^3}{R_{IN}}\right)}{\Delta} \text{ (band pass)} \qquad (28)$$

$$\frac{e_\ell}{e_{IN}} = \frac{-\omega_1\omega_2\left(\frac{2 \times 10^3}{R_{IN}}\right)}{\Delta} \text{ (low pass)} \qquad (29)$$

$$\omega_1 = \frac{10^{12}}{R_{f1} \bullet 220}, \quad \omega_2 = \frac{10^{12}}{R_{f2} \bullet 220}$$

where

$$\Delta = s^2 + s\,\omega_1\left[\frac{1.1 + \frac{2 \times 10^3}{R_{IN}}}{1 + \frac{10^4}{R_Q}}\right] + 0.1\,\omega_1\,\omega_2 \qquad (30)$$

1

Appendix (Continued)

$$\left.\frac{e_\ell}{e_{IN}}\right|_{s \to 0} = \frac{2 \times 10^4}{R_{IN}} \text{ (low pass) (DC gain)} \quad (31)$$

$$\left.\frac{e_h}{e_{IN}}\right|_{s \to \infty} = -\frac{2 \times 10^3}{R_{IN}} \text{ (high pass) (high freq. gain)} \quad (32)$$

$$\left.\frac{e_b}{e_{IN}}\right|_{\omega = \omega_0} = \frac{\frac{2 \times 10^3}{R_{IN}}\left(1 + \frac{10^4}{R_Q}\right)}{1.1 + \frac{2 \times 10^3}{R_{IN}}} \text{ (band pass) (center freq. gain)} \quad (33)$$

$$\omega_0 = \sqrt{0.1\, \omega_1\, \omega_2}$$

$$Q = \left[\frac{1 + \frac{10^4}{R_Q}}{1.1 + \frac{10^4}{R_{IN}}}\right]\sqrt{0.1\frac{\omega_2}{\omega_1}} \quad (34)$$

$$R_Q = \frac{10^4}{\frac{Q}{\sqrt{0.1\frac{\omega_2}{\omega_1}}}\left(1.1 + \frac{2 \times 10^3}{R_{IN}}\right) - 1} \quad (35)$$

d) Differential input *(Figure 14)* transfer function equations are:

$$\frac{e_h}{e_{IN}} = \frac{s^2\left(\frac{2 \times 10^3}{R_{IN2}}\right)}{\Delta} \text{ (high pass)} \quad (36)$$

$$\frac{e_b}{e_{IN}} = \frac{-s\,\omega_1\left(\frac{2 \times 10^3}{R_{IN2}}\right)}{\Delta} \text{ (band pass)} \quad (37)$$

$$\frac{e_\ell}{e_{IN}} = \frac{\omega_1\,\omega_2\left(\frac{2 \times 10^3}{R_{IN2}}\right)}{\Delta} \text{ (low pass)} \quad (38)$$

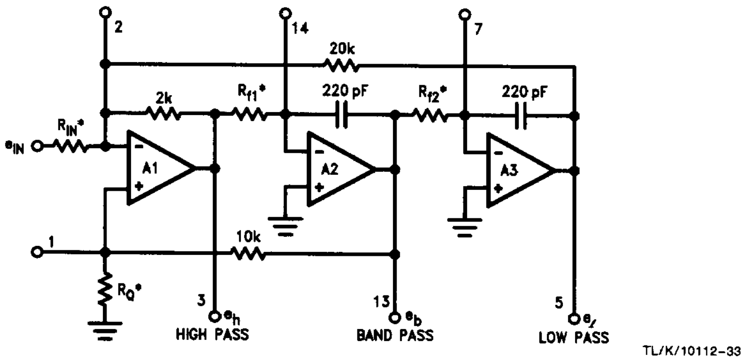
$$\omega_1 = \frac{10^{12}}{R_{f1} \times 220}, \quad \omega_2 = \frac{10^{12}}{R_{f2} \times 220}$$

where

$$\Delta = s^2 + s\,\omega_1\left[\frac{1.1 + \frac{2 \times 10^3}{R_{IN2}}}{1 + \frac{10^4}{R_Q} + \frac{10^4}{R_{IN1}}}\right] + 0.1\,\omega_1\,\omega_2 \quad (39)$$

$$\omega_0 = \sqrt{0.1\,\omega_1\,\omega_2} \quad (40)$$

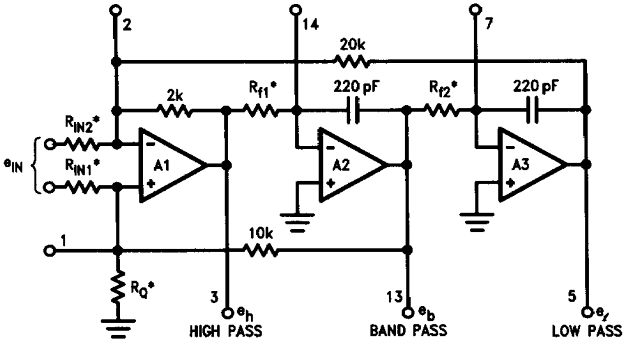
$$Q = \left[\frac{1 + \frac{10^4}{R_Q} + \frac{10^4}{R_{IN1}}}{1.1 + \frac{2 \times 10^3}{R_{IN2}}}\right]\sqrt{0.1\frac{\omega_2}{\omega_1}} \quad (41)$$



*External Components

FIGURE 13. Inverting Input, Any Q

Appendix (Continued)



*External Components

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FIGURE 14. Differential Input

$$R_Q = \frac{10^4}{\frac{Q}{\sqrt{0.1\,\frac{\omega_2}{\omega_1}}}\left(1.1 + \frac{2 \times 10^3}{R_{IN2}}\right) - 1 - \frac{10^4}{R_{IN1}}} \quad (42)$$

e) Notch filter *(Figure 15)* transfer function equations are:

$$\frac{e_n}{e_{IN}} = \frac{(s^2 + \omega_z{}^2)\left[\frac{1.1}{1 + \frac{R_{IN}}{10^4} + \frac{R_{IN}}{R_Q}}\right]\frac{R_g}{R_h}}{s^2 + s\,\omega_1\left[\frac{1.1}{1 + \frac{10^4}{R_Q} + \frac{10^4}{R_{IN}}}\right] + 0.1\,\omega_1\,\omega_2} \quad (45)$$

$$\omega_1 = \frac{10^{12}}{R_{f1} \times 220}, \quad \omega_2 = \frac{10^{12}}{R_{f2} \times 220}, \quad \omega_0 = \sqrt{0.1\,\omega_1\,\omega_2}$$

$$\omega_Z = \omega_0\sqrt{\frac{10\,R_h}{R_L}}$$

$$\left.\frac{e_n}{e_{IN}}\right|_{s \to 0} = \frac{11}{\left(1 + \frac{R_{IN}}{10^4} + \frac{R_{IN}}{R_Q}\right)}\frac{R_g}{R_L} \text{ (DC gain)} \quad (46)$$

$$\left.\frac{e_n}{e_{IN}}\right|_{s \to \infty} = \frac{1.1}{\left(1 + \frac{R_{IN}}{10^4} + \frac{R_{IN}}{R_Q}\right)}\frac{R_g}{R_h} \text{ (high freq. gain)} \quad (47)$$

$$\left.\frac{e_n}{e_{IN}}\right|_{\omega = \omega_Z} = 0 \quad (48)$$

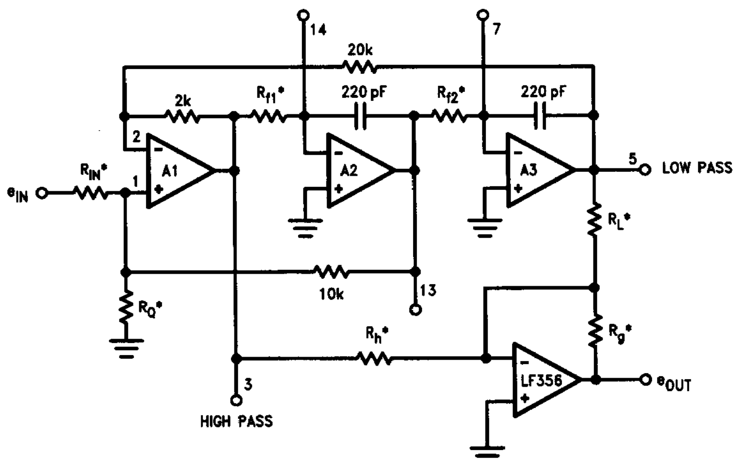

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FIGURE 15. Notch Filter Using an External Amplifier

Appendix (Continued)

f) Input notch filter *(Figure 16)* transfer function equations are:

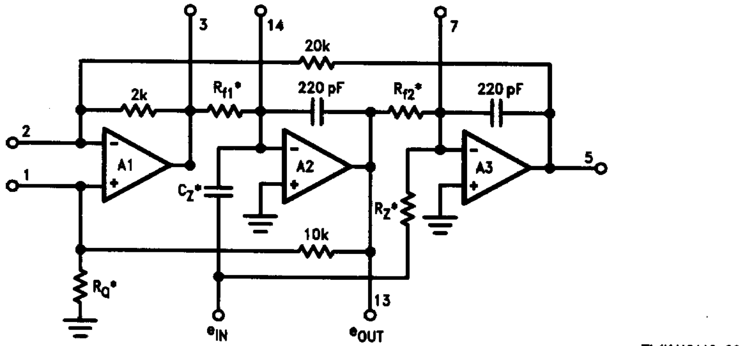
$$\frac{e_{IN}}{e_n} = -\frac{\dfrac{C_Z}{220 \times 10^{-12}}\left[s^2 + \omega_Z{}^2\right]}{s^2 + s\omega_1\left[\dfrac{1.1\,R_Q}{10^4 + R_Q}\right] + \omega_0{}^2} \quad (49)$$

$$\omega_1 = \frac{10^{12}}{R_{f1} \bullet 220}, \quad \omega_2 = \frac{10^{12}}{R_{f2} \bullet 220}$$

$$\omega_Z = \omega_0\sqrt{\frac{R_{f2} \bullet 220 \times 10^{-12}}{R_Z\,C_Z}}, \quad \omega_0 = \sqrt{0.1\,\omega_1\,\omega_2} \quad (50)$$

$$\left.\frac{e_n}{e_{IN}}\right|_{\omega \to 0} = \frac{-R_{F2}}{R_Z} \quad (51)$$

$$\left.\frac{e_n}{e_{IN}}\right|_{\omega \to \infty} = \frac{C_Z}{220 \times 10^{-12}} \quad (52)$$



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*External Components

FIGURE 16. Input Notch Filter Using 3 Amplifiers

g) All pass *(Figure 17)* transfer function equations are:

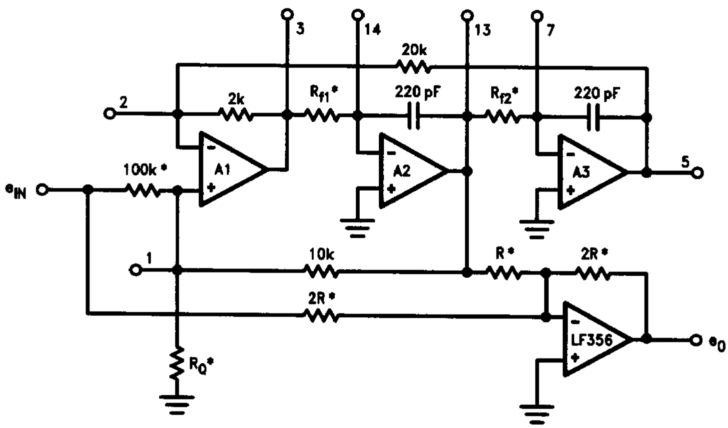
$$\frac{e_o}{e_{IN}} = -\left[\frac{s^2 - s\,\omega_1\left[\dfrac{1.1}{2 + \dfrac{R_{IN}}{R_Q}}\right] + \omega_0{}^2}{s^2 + s\,\omega_1\left[\dfrac{1.1}{2 + \dfrac{R_{IN}}{R_Q}}\right] + \omega_0{}^2}\right] \quad (53)$$

$$Q = \left[\frac{2 + \dfrac{10^4}{R_Q}}{1.1}\right]\sqrt{0.1\,\frac{\omega_2}{\omega_1}} \quad (54)$$

$$\omega_1 = \frac{10^{12}}{R_{f1} \bullet 220}, \quad \omega_2 = \frac{10^{12}}{R_{f2} \bullet 220}$$

$$\omega_0 = \sqrt{0.1\,\omega_1\,\omega_2}$$

Time delay at ω_0 is $\dfrac{2Q}{\omega_0}$ seconds



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*External Components

FIGURE 17. All Pass

Definition of Terms

A_{MAX}	Maximum pass band peak-to-peak ripple
A_{MIN}	Minimum stop band loss
f_z	Frequency of jw axis pole pair
f_o	Frequency of complex pole pair
Q	Quality of pole
f_c	Pass band edge
f_s	Stop band edge
R_f	Pole frequency determining resistance
R_z	Zero Frequency determining resistance
R_Q	Pole quality determining resistance
f_H	Frequency above center frequency at which the gain decreases by 3 dB for a band pass filter
f_L	Frequency below center frequency at which the gain decreases by 3 dB for a band pass filter

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